The point is, therefore, the center of mean position of the n points as stated.

Also solved by J. Scheffer.

264. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

The join of the center of curvature of a curve to the origin is at α to the initial line. Prove that with the usual notation:

$$\frac{d\alpha}{d\psi}\left[\left(\frac{dp}{d\psi}\right)^2+\left(\frac{d^2p}{d\psi^2}\right)^2\right]=\frac{dp}{d\psi}\cdot\frac{d\rho}{d\psi}.$$

No solution of this problem has been received.

265. Proposed by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Find two curves which possess the property that the tangents TP and TQ to the inner one always make equal angles with the tangent TT' to the outer.

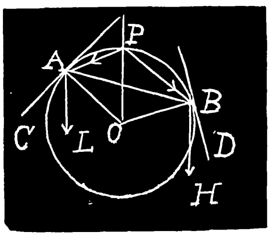
No solution of this problem has been received.

MECHANICS.

219. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

A rod length $a\sqrt{3}$, weight W, has at each end a smooth ring which can slide on a vertical circle radius r. Each ring is attached by an elastic string (natural lengths a, b; moduli μa, μb) to the highest point of the circle. Find the inclination of the rod to the horizon in a position of equilibrium.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.



In what follows we regard the strings as having no weight, and also that both strings are in tension from the weight of the rod and rings, and the rod is above the center of the circle. Let $AB=a\sqrt{3}=$rod; $AP=$string a; $BP=$string b; O, the center of the circle, radius $AO=r$. Draw AK perpendicular to PO. Let $m=$ weight of each ring; $\angle AOB=\beta=2\sin^{-1}(a\sqrt{3}/2r)$; $\angle APB=\pi-\frac{1}{2}\beta$; $\angle KAB=\theta=$angle AB makes with the horizon; $\angle PAB=\phi$; $\angle PBA=\frac{1}{2}\beta-\phi$; $T=$tension of string AP; $T'=$tension of string PB. When in equilibrium, W and the components of T, T' tangent at A, B meet in a point. $AP=a(1+T/\mu a)=(\mu a+T)/\mu$, $BP=(\mu b+T')/\mu$, $T=(\frac{1}{2}W+m)\sin(\phi-\theta)$. Let $(\frac{1}{2}W+m)=Q$.

$\therefore T=Q\sin(\phi-\theta)$, $T'=Q\sin(\theta+\frac{1}{2}\beta-\phi)$, $AP/PB=(\mu a+T)/(\mu b+T')=\sin(\frac{1}{2}\beta-\phi)/\sin\phi$...(1).

$3\mu^2a^2=(\mu a+T)^2+(\mu b+T')^2+2(\mu a+T)(\mu b+T')\cos\frac{1}{2}\beta$...(2).

The values of T and T' in (1) and (2) give

$[\mu a+Q\sin(\phi-\theta)]\sin\phi=[\mu b+Q\sin(\theta+\frac{1}{2}\beta-\phi)]\sin(\frac{1}{2}\beta-\phi)$...(3).

$$3\mu^2a^2=[\mu a+Q\sin(\phi-\theta)]^2+[\mu b+Q\sin(\theta+\tfrac{1}{2}\beta-\phi)]^2$$
$$+2[\mu a+Q\sin(\phi-\theta)][\mu b+Q\sin(\theta+\tfrac{1}{2}\beta-\phi)]\cos\tfrac{1}{2}\beta\ldots(4).$$

Eliminating ϕ between (3) and (4) gives an equation to determine θ.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

152. Proposed by H. S. VANDIVER, Bala, Pa.

When p is a prime of the form $5n\pm1$ then there is a positive integer a such that $a^2\equiv5(\text{mod }p)$. Show that $\left(\dfrac{a\pm1}{p}\right)=\pm\left(\dfrac{-2a}{p}\right)$, according as p is of the form $5n+1$ or $5n-1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A general expression involving all cases is, apparently, not easily deduced. The following cases hold.

(1) Let $n=2$. Then $p=5n+1=11$. $a=4$, $a=p-4=7$, $a=2p-7=15$, $a=3p-15=18$, etc.

(2) Let $n=4$. Then $p=5n-1=19$, $a=9$, $a=p-9=10$, $a=2p-10=28$, $a=3p-28=29$, etc.

(3) Let $n=6$. Then $p=5n+1=31$, $a=6$, $a=p-6=25$, $a=2p-25=37$, $a=3p-37=56$, etc.

(4) Let $n=8$. Then $p=5n+1=41$, $a=13$, $a=p-13=28$, $a=2p-28=54$, $a=3p-54=69$, etc.

(5) Let $n=12$. Then (b) $p=5n+1=61$; (c) $p=5n-1=59$.

(b) $a=26$, $a=p-26=35$, $a=2p-35=87$, $a=3p-87=96$, etc.

(c) $a=8$, $a=p-8=51$, $a=2p-51=67$, $a=3p-67=110$, etc.

For every value of n that makes $5n\pm1$ a prime, we can find values for a satisfying the condition. It is also easy to see that a can have an infinitude of values for each case.

In (1), (3), (4), (5)(b), $(a+1)^{\frac{1}{2}(p-1)}=(-2a)^{\frac{1}{2}(p-1)}\equiv-1(\text{mod. }p)$.

In (2), (5)(c), $(a-1)^{\frac{1}{2}(p-1)}=-(-2a)^{\frac{1}{2}(p-1)}\equiv1(\text{mod. }p)$.

$\left(\dfrac{a\pm1}{p}\right)=\pm\left(\dfrac{-2a}{p}\right)$, in the cases examined, according as $p=5n\pm1$.

A general solution is desired. ED. F.

152. Proposed by H. S. VANDIVER, Bala, Pa.

Prove geometrically:

$$\sum_{n=1}^{\frac{1}{2}(p-1)}\left[\frac{n^2}{p}\right]=\frac{p-3}{4}\cdot\frac{p-1}{2}-\sum_{n=1}^{\frac{1}{4}(p-4}\left[\sqrt{np}\right],$$ where $p\equiv3(\text{mod. }4)$ and $\left[\dfrac{k}{p}\right]$ represents the greatest integer in k/p.